Non-stationary effective parameters for distributed hydrological modeling

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1. INTRODUCTION

Hydrological modeling at watershed scale is highly affected by uncertainty as a consequence of simplifications in physical processes representation, errors in the measurement of variables, errors in parameters estimation and the hardness of initial and boundary conditions estimation (Rosbjerg and Madsen 2005; Gourley and Vieux 2006). It is important for distributed hydrological modeling to assess the sources of uncertainty and reduce its influence.

The main aim of this work is to test the importance of representing sub-grid heterogeneities of soil parameters within a distributed hydrological modeling framework. It was compared the difference of taking into account sub-grid spatial heterogeneities effect by means of Monte-Carlo simulations, semi-empirical equations with non-stationary effective parameters and effective parameters.

2. MODEL DESCRIPTION

It was used the conceptualization of the hydrological model called TETIS, which is a spatially distributed model. Runoff production is modeled at each grid cell by means of six conceptual tanks. Fluid flowing through tanks represents precipitation, snowmelt, evapotranspiration, infiltration, percolation, groundwater outflow, overland flow, interflow and base flow (figure 1). A detailed description of model's development is presented by Francés et. al. (2007).

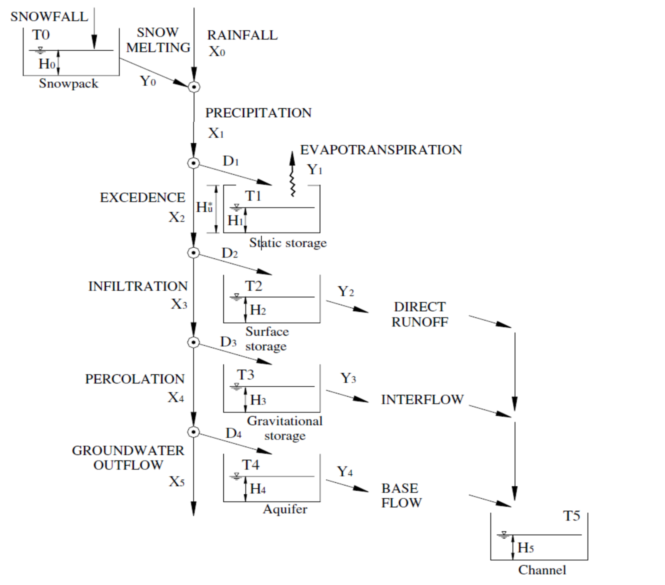

Figure 1. Scheme of the TETIS model (Francés et. al, 2007).

We focus our research on sub-grid variability of infiltration and percolation processes. The static storage tank represents the upper soil capillary water storage and has a threshold corresponding to "H_u". Precipitation X_1 is accumulated into this tank when Hu is not reached. We represent the excess water by [1] and capillary infiltration by [2].

Once the static storage tank reaches Hu, the infiltration capacity has a value near to upper soil saturated hydraulic conductivity k_s and it is reasonable to express gravitational infiltration X_3 by [3].

that total infiltration will be the sum of D_1 and X_3. The gravitational infiltration could join to gravitational storage or go downwards as percolation flow X_4. The percolation capacity is approximated by the deep soil saturated hydraulic conductivity k_p, and X_4 will be calculated similarly to equation 3.

$$X_2 = Max\ (\ 0\ ;\ X_1 - H_u + H_1) \quad [1]$$

$$D_1 = X_1 - X_2 \quad [2]$$

$$X_3 = Min\ (\ X_2\ ;\ \Delta t \quad k_s\ ) \quad [3]$$

3. METHODS

Comparing four approaches to consider parameter's sub-grid heterogeneity in Goodwin Creek experimental watershed:

App1: Using the parameter values estimated at the support of cartographic units and calibrating a correction factor.

App2: By an explicit representation using Monte Carlo simulations. Assuming beta distribution of H_U [Beta(2,2)], and lognormal distribution of k_s and k_p we generated random fields of each parameter at sub-grid support. Flow equations were solved at sub-grid support and then aggregated at grid support. Calibrating a correction factor related to the expected value at grid support.

App3: Following the assumptions of *App2*, it was derived probability distribution functions analytically for state and flow variables. Then we calculate expected value equations for flow at grid support. By inverse problem solving were found equations for H_u, k_s and k_p at grid support, which depends on input and state variables. (denoted as non-stationary effective parameters - equations [4] to [12]).

App4: Using the parameter values estimated at grid support and calibrating a correction factor.

For random fields of iid random variables:

$$f_{H_{1f}}\left(H_{1f}\right)=\begin{cases}\dfrac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)}\left(\dfrac{H_u}{\Lambda}\right)^{a-1}\left(1-\dfrac{H_u}{\Lambda}\right)^{b-1} & \text{if}\quad H_1+X_1>H_u\\ \dfrac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)}\left(\dfrac{H_1}{\Lambda\cdot w}\right)^{a-1}\left(1-\dfrac{H_1}{\Lambda\cdot w}\right)^{b-1} & \text{if}\quad H_1+X_1\le H_u\end{cases} \quad [4]$$

$$E\left[H_{1f}\right]=H_{uEF}=\int_0^{x_1/(1-w)}\frac{H_u}{\Lambda}\cdot\frac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)}\cdot\left(\frac{H_u}{\Lambda}\right)^{a-1}\left(1-\frac{H_u}{\Lambda}\right)^{b-1}dH_u+\int_{x_1/(1-w)}^{\Lambda w+X_1}\frac{H_1}{\Lambda}\cdot\frac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)}\cdot\left(\frac{H_1-X_1}{\Lambda}\right)^{a-1}\left(1-\frac{H_1-X_1}{\Lambda}\right)^{b-1}dH_1 \quad [5]$$

$$H_{uEF,t}=\begin{cases}\dfrac{l_t^3\left(4\Lambda-3l_t\right)}{2\Lambda^3}-\dfrac{m_t^2\left[6n_t^2-8n_tm_t+3m_t^2\right]+\Lambda w m_t^2\left[6n_t-4m_t\right]}{2(\Lambda w)^3}+\dfrac{l_t^2\left[6n_t^2-8n_tl_t+3l_t^2\right]+\Lambda w l_t^2\left[6n_t-4l_t\right]}{2(\Lambda w)^3} & \text{if}\quad n_t<\Lambda\\ \dfrac{\Lambda}{2} & \text{if}\quad n_t\ge\Lambda\end{cases} \quad [6]$$

$$n_t=n_{t-1}+X_{1,t} \qquad l_t=\frac{n_t}{(1-w)} \qquad m_t=\Lambda w+n_t$$

$$F_{X_3}\left[X_3\right]=1-\left[1-F_{k_s}\left(X_3/\Delta t\right)-F_{X_2}\left(X_3\right)+F_{k_s}\left(X_3/\Delta t\right)\cdot F_{X_2}\left(X_3\right)\right] \quad [7]$$

$$f_{X_3}\left[X_3\right]=F'_{k_s}\left(X_3/\Delta t\right)+F'_{X_2}\left(X_3\right)-F'_{k_s}\left(X_3/\Delta t\right)\cdot F_{X_2}\left(X_3\right)-F_{k_s}\left(X_3/\Delta t\right)\cdot F'_{X_2}\left(X_3\right) \quad [8]$$

$$E\left[X_{3,t}\right]=k_{sEF,t}=\int_0^{X_{3,t}\max}X_{3,t}\cdot f_{X_{3,t}}\left(X_{3,t}\right)dX_{3,t} \quad [9]$$

The derivation of $k_{pEF,t}$ has a similar structure to equations [7], [8] and [9].

When it is considered the spatial autocorrelation of parameter fields is hard to conditioning each random variable on the others. Specially dealing with a large number of them. So, we found that the non-stationary effective parameters H_u, k_s and k_p can be expressed by:

$$H_{uef,t}=(X_{1(t)}+H_{1(t)})\left\{1-\Phi\left[\frac{\ln(X_{1(t)}+H_{1(t)}-\mu}{\sigma}\right]\right\}+\overline{H_u}\quad\left\{\Phi\left[\frac{\ln(X_{1(t)}+H_{1(t)})-\mu}{\sigma}-\omega_1\mu^{\omega_2}\sigma\right]\right\} \quad [10]$$

$$k_{sef,t}=\overline{k_s}\quad\left\{1-\varepsilon\left(X_{2(t)},\alpha\quad\sigma(k_s)\right)\right\}-X_{2(t)}\quad\left\{\varepsilon\left(X_{2(t)},\alpha\quad\sigma(k_s)\right)\right\} \quad [11]$$

$$k_{pef,t}=\overline{k_p}\quad\left\{1-\varepsilon\left(X_{3(t)},\alpha\quad\sigma(k_p)\right)\right\}-X_{3(t)}\quad\left\{\varepsilon\left(X_{3(t)},\alpha\quad\sigma(k_p)\right)\right\} \quad [12]$$

4. CASE STUDY

CALIBRATION AND VALIDATION:

Calibration was carried out by means of the SCE-UA optimization algorithm at the outlet gauge station. Each approach was applied with a spatial resolution of 30x30 m² and temporal discretization of 5 min.

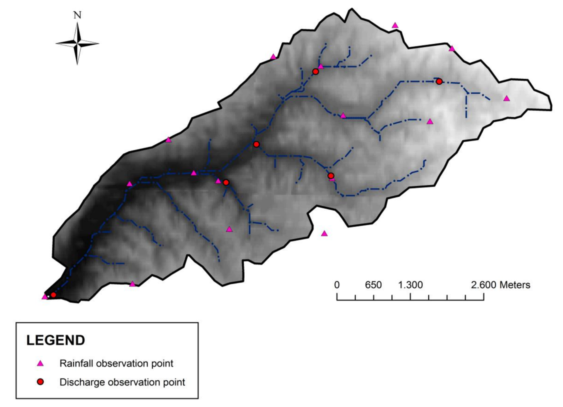

Figure 2. Goodwin Creek experimental watershed.

Event		Total rainfall (mm)	Duration (hours)
1	Calibration	149.11	35.08
2	Validation	74.22	14.83
3	Validation	148.28	60.17
4	Validation	44.00	30.66
5	Validation	61.56	25.33

Table 1. rainfall events for calibration and validation

5. RESULTS

The spatial-temporal validations indicate an important difference in model performance when comparing the four approaches (Figure 5). There is a tendency to improve the agreement between observations and simulations when is considered the sub-grid heterogeneity by Monte-Carlo simulations (App 2) and non-stationary effective parameters (App 3) for low rainfall events.

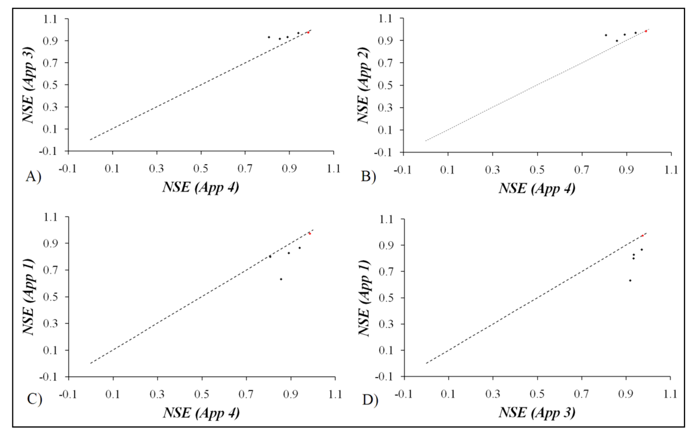

Figure 3. Comparison of Nash-Sutcliffe efficiencies (NSE) in calibration (red points) and spatial validation (black points) for all approaches.

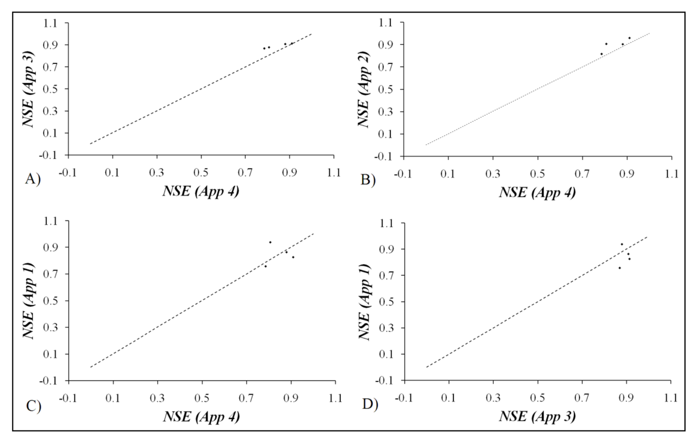

Figure 4. Contrast of NSE in temporal validation for all approaches.

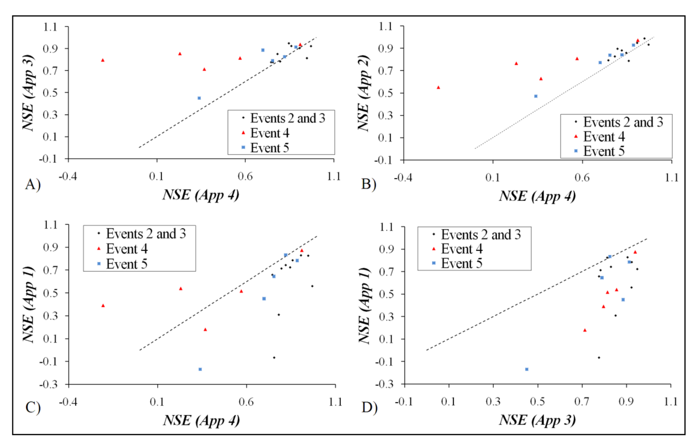

Figure 5. Comparison of NSE in spatial-temporal validation for all approaches.

6. CONCLUSIONS

Based on that results, we reflect on that sub-grid heterogeneity of parameters is an essential subject in hydrological modeling. The main advantage of taking into account sub-grid heterogeneity is that we can obtain a more robust calibrated hydrological model than using stationary effective parameters. The robustness is improved in the sense of better performance of runoff simulations for low magnitude rainfall events.

Nevertheless, the stationary effective parameters have shown a good representation of watershed properties for runoff modeling and its results are close to sub-grid results in high magnitude events.

In this work we didn't test model performance in terms of simulating the state variables, which is relevant to advance the knowledge of sub-grid heterogeneities effect in the hydrological model. It should be studied in the near future to support the above conclusions.

Our study focused on event driven approach, which is high sensitive to initial conditions in the watershed. To minimize the error by initial condition estimation, we consider necessary to carry out a similar work by continuous simulation.

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REFERENCES

Francés, F., Vélez, J.I., Vélez, J.J. (2007) Split-parameter structure for the automatic calibration of distributed hydrological models. *Journal of Hydrology* **332**: 226-240.

Gourley, J. J. and B. E. Vieux (2006). "A method for identifying sources of model uncertainty in rainfall-runoff simulations." *Journal of Hydrology* **327** (1-2): 68-80.

Rosbjerg, D. and H. Madsen (2005). Concepts of hydrologic modelling. Encyclopedia of hydrological sciences. M. G. Anderson and J. J. McDonnell. Chichester, Reino Unido, John Wiley & Sons: 155-163.