Parameterization of subgrid heterogeneities for hydrologic modelling

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1. Introduction

Soil heterogeneity is relevant for hydrological processes modelling and distributed hydrological models take into account that issue by an explicit representation of spatial variability. Parametric and mechanistic models are characterized by de problem that its parameter's values depend on the scale at which they are calibrated. Going downwards in model development we need to know what level of complexity is sufficient to represent the main behavior of our hydrological systems.

We present a formulation to parameterize subgrid heterogeneities of soil hydraulic parameters and its incorporation within TETIS hydrological model. The parameterization approach was tested in a small experimental watershed and compared with TETIS parameterization without subgrid representation by using aggregated parameters at a coarser support. The representation of subgrid heterogeneities improved model performance in spatial-temporal validation.

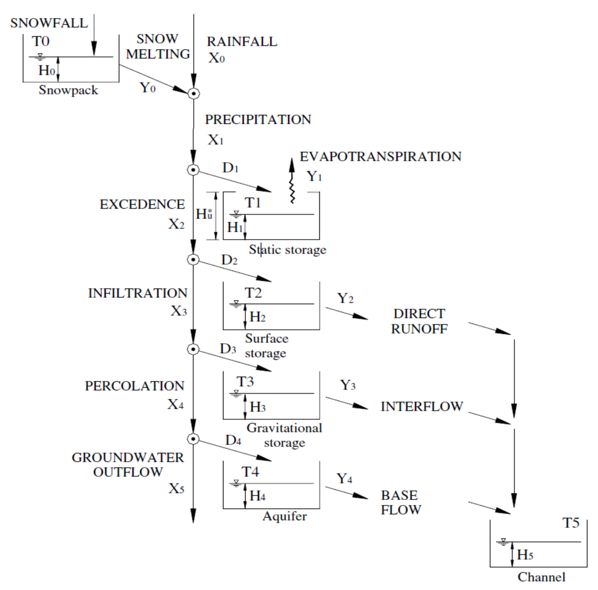

Figure 1. Scheme of TETIS Hydrological model (Francés et. al, 2007)

2. Approach

Based on the assumption of Beta distribution of soil static storage capacity (H_u) and Lognormal distribution of saturated hydraulic conductivities k_s and k_p at point scale (S1) we calculated derived distribution functions of flow variables at point scale. We propose the inversion of the Equations (1 and 2) to calculate non-stationary effective parameters at cell scale (S2). This technique lets us to calculate H_u[S2] when X_2[S2] is larger than zero (Equation 3), that assumption preserves mass balance. The inversion of Equation 2 lets us to state that the parameter k_s[S2] is equal to X_3[S2], and a similar procedure was carried out to calculate effective parameters of k_p. The resulting equations are expressed in an integral form and the validity of those equations was tested by Monte Carlo simulations on a single cell containing N subcells.

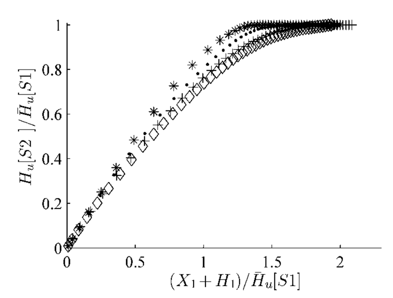

Figure 2. Non-stationary effective parameter as a function of input and state variable.

We formulated semi-empirical equations of non-stationary effective parameters because of the analytical ones must be solved by numerical integration and their solution would require the use of many computational resources to be applied in a real case study.

3. Results

Direct Formulation:

$$X_2 = Max\,(0\,;\,X_1 - H_u + H_1) \quad [1]$$

$$X_3 = Min\,(X_2\,;\,\Delta t \; k_s) \quad [2]$$

Inverse Formulation:

$$H_{u,t}[S2] = X_{1,t}[S2] + H_{1,t}[S2] - X_{2,t}[S2] \quad [3]$$

$$k_{s,t}[S2] = \begin{cases} X_{2,t}[S2]\cdot(\Delta t)^{-1} & X_{3,t}[S2] = X_{2,t}[S2] \\ X_{3,t}[S2]\cdot(\Delta t)^{-1} & X_{3,t}[S2] < X_{2,t}[S2] \end{cases} \quad [4]$$

Analytical non-stationary effective parameters:

$$f_{H_{1f}}(H_{1f}) = \begin{cases} \dfrac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)}\left(\dfrac{H_u}{\Lambda}\right)^{a-1}\left(1-\dfrac{H_u}{\Lambda}\right)^{b-1} & \text{if } H_1 + X_1 > H_u \\ \dfrac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)}\left(\dfrac{H_1}{\Lambda\cdot w}\right)^{a-1}\left(1-\dfrac{H_1}{\Lambda\cdot w}\right)^{b-1} & \text{if } H_1 + X_1 \le H_u \end{cases} \quad [5]$$

$$E[H_{1f}] = H_{uEF} = \int_0^{X_1/(1-w)} \frac{H_u}{\Lambda}\cdot\frac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)}\cdot\left(\frac{H_u}{\Lambda}\right)^{a-1}\left(1-\frac{H_u}{\Lambda}\right)^{b-1} dH_u + \int_{X_1/(1-w)}^{\Lambda w + X_1} \frac{H_1}{\Lambda}\cdot\frac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)}\cdot\left(\frac{H_1 - X_1}{\Lambda}\right)^{a-1}\left(1-\frac{H_1 - X_1}{\Lambda}\right)^{b-1} dH_1 \quad [6]$$

$$H_{uEF,t} = \begin{cases} \dfrac{l_t^3(4\Lambda - 3l_t)}{2\Lambda^3} - \dfrac{m_t^2\left[6n_t^2 - 8n_t m_t + 3m_t^2\right] + \Lambda w m_t^2\left[6n_t - 4m_t\right]}{2(\Lambda w)^3} + \dfrac{l_t^2\left[6n_t^2 - 8n_t l_t + 3l_t^2\right] + \Lambda w l_t^2\left[6n_t - 4l_t\right]}{2(\Lambda w)^3} & \text{if } n_t < \Lambda \\ \dfrac{\Lambda}{2} & \text{if } n_t \ge \Lambda \end{cases} \quad [7]$$

$$n_t = n_{t-1} + X_{1,t} \qquad l_t = \frac{n_t}{(1-w)} \qquad m_t = \Lambda w + n_t$$

$$F_{X_{2,t}}[X_{2,t}] = \begin{cases} 1 & \text{if } X_{1,t} \le X_{2,t} \\ \dfrac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)}\displaystyle\int_0^{p_t/\Lambda} \tau^{a-1}(1-\tau)^{b-1}\,d\tau & \text{if } H_u \ge H_{1,t} + X_{1,t} \\ \dfrac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)}\cdot\left(\dfrac{q_t^a}{a(1-w)^a\Lambda^a}\right)\cdot {}_2F_1\left(a, 1-b; a+1; \dfrac{q_t}{(1-w)\Lambda}\right) & \text{in otherwise} \end{cases} \quad [8]$$

$$p_t = \frac{(1-w)\Lambda - n_t}{(1-w)} \qquad q_t = X_{2,t} + (1-w)\Lambda - n_t \qquad {}_2F_1 = \text{Hypergeometric function}$$

$$F_{X_3}[X_3] = 1 - \left[1 - F_{k_s}(X_3/\Delta t) - F_{X_2}(X_3) + F_{k_s}(X_3/\Delta t)\cdot F_{X_2}(X_3)\right] \quad [9]$$

$$f_{X_3}[X_3] = F'_{k_s}(X_3/\Delta t) + F'_{X_2}(X_3) - F'_{k_s}(X_3/\Delta t)\cdot F_{X_2}(X_3) - F_{k_s}(X_3/\Delta t)\cdot F'_{X_2}(X_3) \quad [10]$$

$$E[X_{3,t}] = k_{sEF,t} = \int_0^{X_{3,t}\max} X_{3,t}\cdot f_{X_{3,t}}(X_{3,t})\,dX_{3,t} \quad [11]$$

Calculation of $k_{pEF,t}$ has a similar structure to equations [7], [8] and [9].

Semi-empirical equations of non-stationary effective parameters:

$$H_{uef,t} = (X_{1(t)} + H_{1(t)})\left\{1 - \Phi\left[\frac{\ln(X_{1(t)} + H_{1(t)} - \mu}{\sigma}\right]\right\} + \overline{H_u}\left\{\Phi\left[\frac{\ln(X_{1(t)} + H_{1(t)}) - \mu}{\sigma} - \omega_1 \mu^{\omega_2}\sigma\right]\right\} \quad [12]$$

$$k_{sef,t} = \overline{k_s}\left\{\varepsilon\left(X_{2,t}, \alpha\cdot\sigma(k_s)\right)\right\} - X_{2,t}\left\{1 - \varepsilon\left(X_{2,t}, \alpha\cdot\sigma(k_s)\right)\right\} \quad [13]$$

$$k_{pef,t} = \overline{k_p}\left\{\varepsilon\left(X_{3,t}, \alpha\cdot\sigma(k_p)\right)\right\} - X_{3,t}\left\{1 - \varepsilon\left(X_{3,t}, \alpha\cdot\sigma(k_p)\right)\right\} \quad [14]$$

Verification by Monte Carlo simulations:

realizations	*10000*
a	*2*
b	*2*
Λ	*20*
mean k_s	*5*
variance k_s	*100*
mean k_p	*6*
variance k_p	*144*
dt	*1/6 hours*
H1 initial	*0.2*Hu*
alpha k_s	*0.0996*
apha k_p	*0.104*
mu	*2.306*
sigma	*5.82E-04*

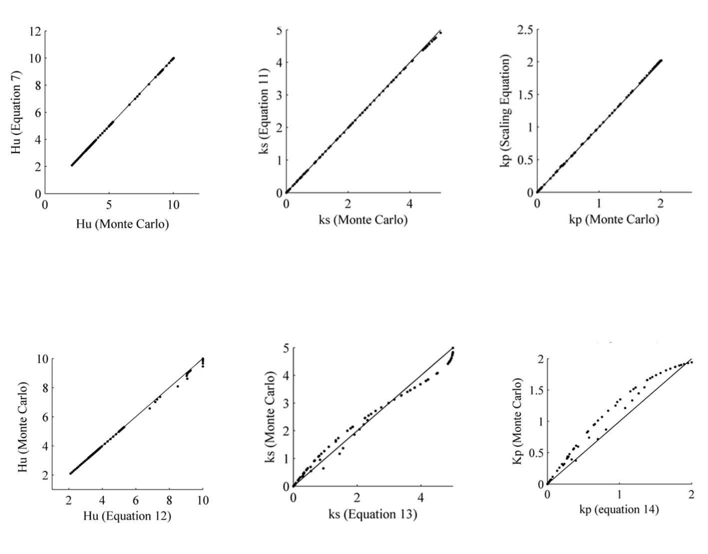

Figure 3. Scatter plots showing the validity of non-stationary effective parameters equations.

MODEL CALIBRATION

Calibration procedure was carried out for a rainfall-runoff event (19-09-1983). The SCE-UA optimization algorithm was used to maximize Nash-Sutcleffe Eficiency at the outlet gauge station. Three calibrations were performed: 1. parameter values resolution of 30x30 (Support 1), 2. parameter values resolution of 1732x1732 (Support 2) and 3. the coarser parameter resolution with subgrid equations (Support 2 + subgrid).

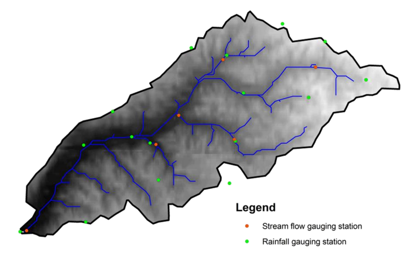

Figure 4. Study area.

Each calibration was applied with a spatial resolution of 30x30 m^2 and temporal discretization of 5 minutes.

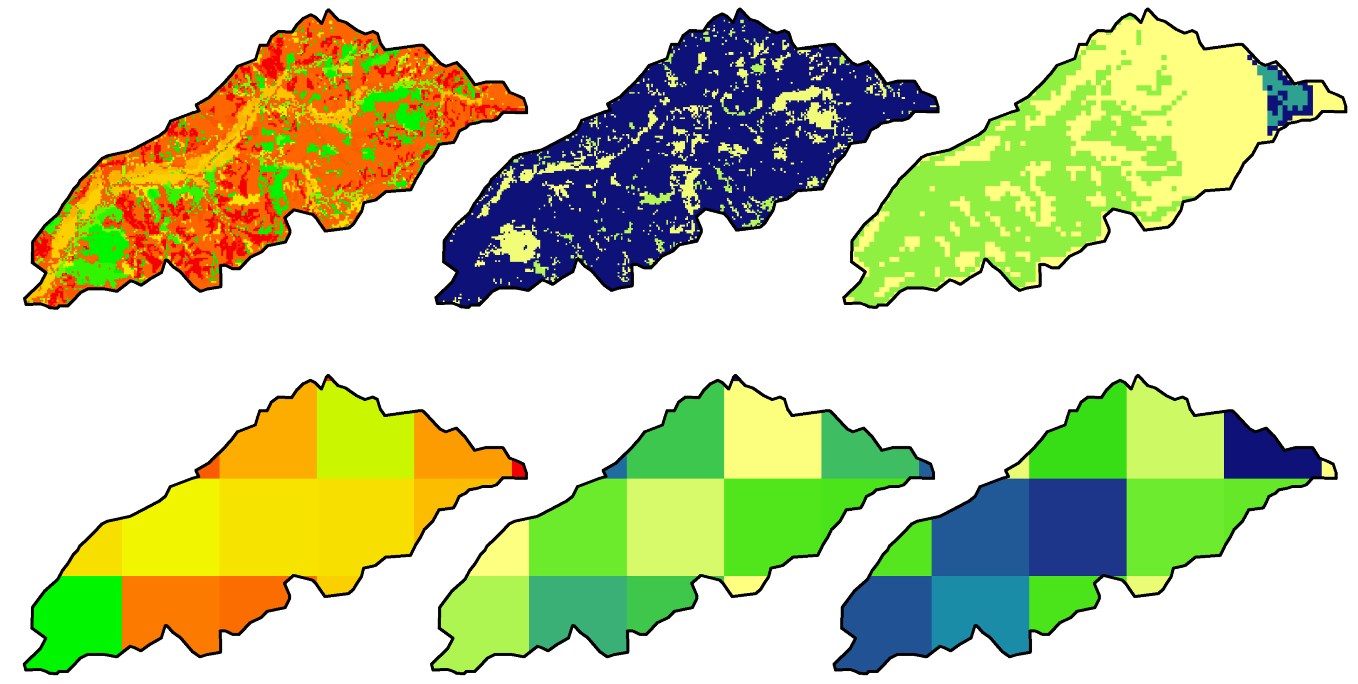

Figure 5. Parameter fields used to test the utility of subgrid representation. Top: 30x30 m^2 spatial resolution (Support 1). Down: 1732x1732 m^2 of spatial resolution (Support 2) .

VALIDATION

1. The spatial-temporal validations indicate an important difference in model performance when comparing the three approaches (Figure 6). There is a consistent improvement of model performance when is considered the sub-grid heterogeneity by non-stationary effective parameters.

2. Validation by continuous simulation: The validation for an entire year (1984) confirms the better performance of the hydrological model by using equations taking into account subgrid heterogeneities. The worst performance was founded neglecting subgrid variability and using the coarser spatial resolution of parameters.

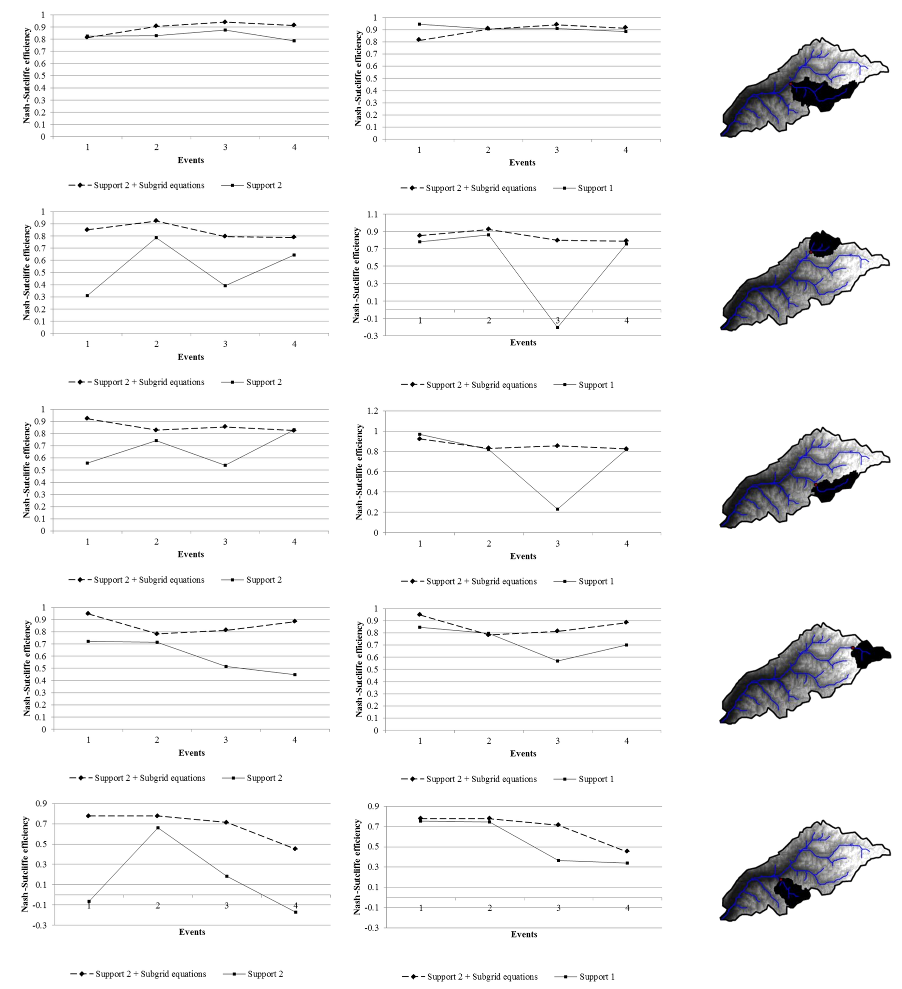

Figure 6. Comparison of NSE in spatial-temporal validation for all approaches.

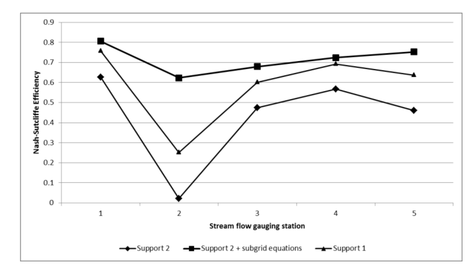

Figure 7. Comparison of NSE in spatial-temporal validation for continuous simulation (year 1984).

4. Conclusions

This work present the development of scaling equations to transfer the effect of subgrid spatial heterogeneities of soil parameters within a hydrological model. The implementation of hose equations in TETIS model demonstrated its potential to improve the representation of the hydrological processes within the catchment.

The main advantage of taking into account sub-grid heterogeneity is that we can obtain a more robust calibrated hydrological model than using stationary effective parameters. The robustness is improved in the sense of better performance of runoff simulations at locations don't used to hydrological model. calibration

Nevertheless, the stationary effective parameters have shown a good representation of watershed properties for runoff modeling and its results are close to sub-grid results in high magnitude events.

The results of validation by continuous simulation confirms the utility of subgrid equations to represent the Hydrology of Goodwin Creek. But it is needed to contrast this hypothesis in other catchments to state a stronger analysis based on the study of a wide range of hydrologic conditions.

Acknowledges

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