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IMPROVING THE BAYESIAN JOINT INFERENCE THROUGH THE INCLUSION OF HYDROLOGICAL STATE VARIABLES IN THE RESIDUALS DEPENDENCE MODEL

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Introduction (1/3)

- **Problem:** Hydrological models provide predictions, which are not lacking of uncertainty
  - In general, model state variables(e.g. streamflow "q_s") do not match observations of the predictand "q" $q \neq q_s$
- Considering
  - "q_s" as a Random Variable
  - The existence of **the joint pdf** $p\left(q, q_s(\theta_{h,e}; \tilde{X}_n)\right)$

  - We can define the **Predictand pdf conditioned on q_s (Predictand cpdf)** $p\left(q \middle| q_s\right)$
- So far, equations are independent of the kind of error model (additive/multiplicative)
  - If we consider an **additive error**, **Predictand cpdf = Error cpdf**

$$q = q_s + e \quad \rightarrow \quad p\left(q \middle| q_s\right) = p\left(e \middle| q_s\right)$$

Introduction (2/3)

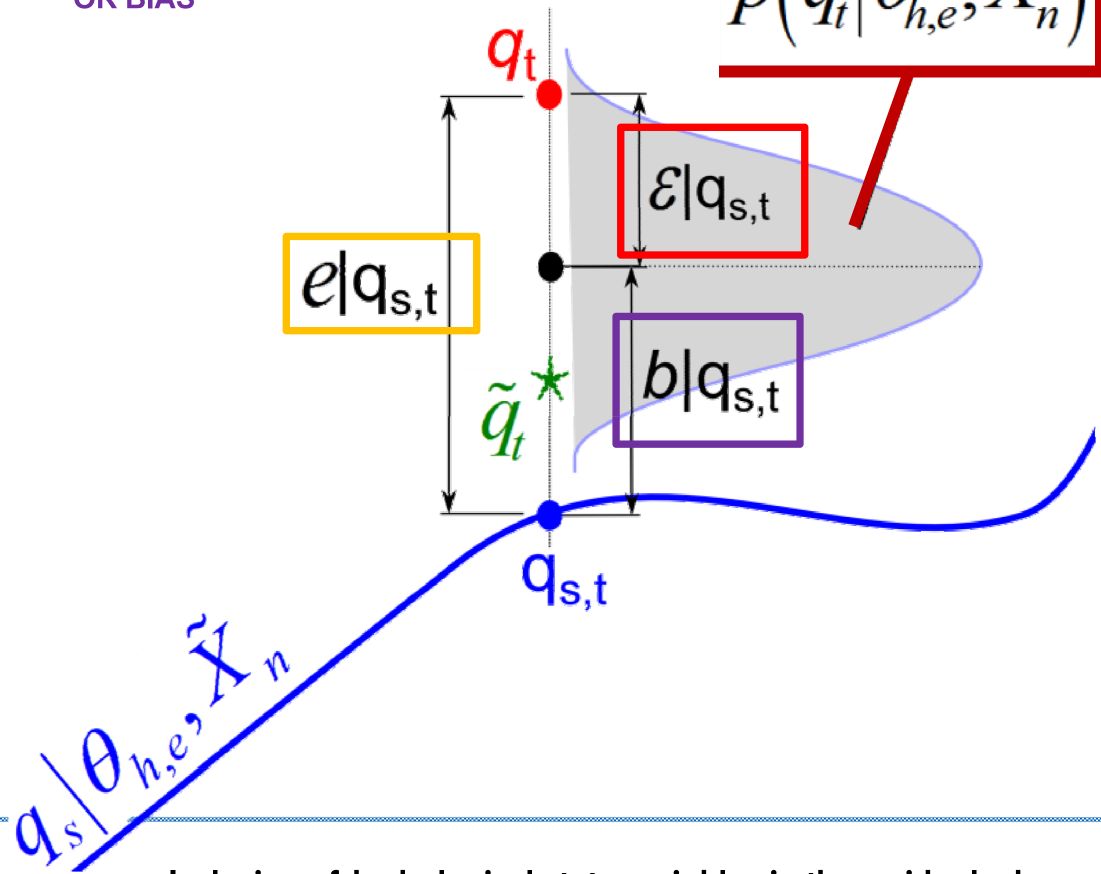
- **Modeling the Error term** $q = q_s + e$
  - We can model the **two components of Error** at time step "t"

$$q_t = q_{s,t} + b|q_{s,t} + \varepsilon|q_{s,t} \rightarrow p\left(q_t|q_{s,t}\right) = p\left(e|q_{s,t}\right) = p\left(\varepsilon|q_{s,t}\right)$$

DETERMINISTIC OR BIAS RANDOM ERROR

$$p\left(q_t|\theta_{h,e},\tilde{X}_n\right)$$

THE PREDICTIVE CPDF



Introduction (3/3)

- **Classical approach for modeling the Error term**
  - Considers additive errors serially uncorrelated (White Noise)
  - With Gaussian distribution
  - Constant conditional variances (homoscedastic errors)
  - It does not account for Bias

UNBIASED I.I.D. ERROR

➢ **Equivalent to Std. Least Squares calibration (SLS)**

- **Errors in Hydrology** do not satisfy the **SLS** hypotheses
  - **Causes** are mainly the **Input errors** and an unsuitable **H. Model structure**
  - **Consequences**
    - Biased or "corrupted" **parameter values**
    - An incorrect estimation of the **Predictive uncertainty**

Aims of this research

- **Phase I**
  - Inferring a **Specific Error Model** that best fits Hydrological Model Errors
    - **Inference must be a JOINT INFERENCE to avoid Biased parameters in both models**
  - Compare Performance of **SLS vs Specific Error Model**

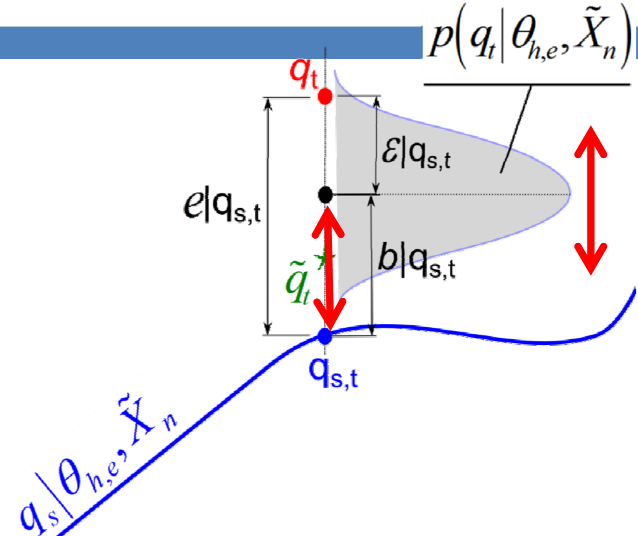
- **Phase II** (Not concluded)
  - We try an upgrade of the error model through a **Bias model improvement**, in order to achieve a better performance than in phase I

Phase I: Error Model description

Time-varying Error variance & Bias

Variance $\sigma_{\varepsilon|q_{s,t}} = \theta_1^e + \theta_2^e q_{s,t}$

Bias $\begin{cases} b_{e|q_{s,t}} = \theta_3^e + \theta_4^e q_{s,t} & q_{s,t} \le \theta_5^e \\ b_{e|q_{s,t}} = \theta_6^e + \theta_7^e q_{s,t} & q_{s,t} > \theta_5^e \end{cases}$



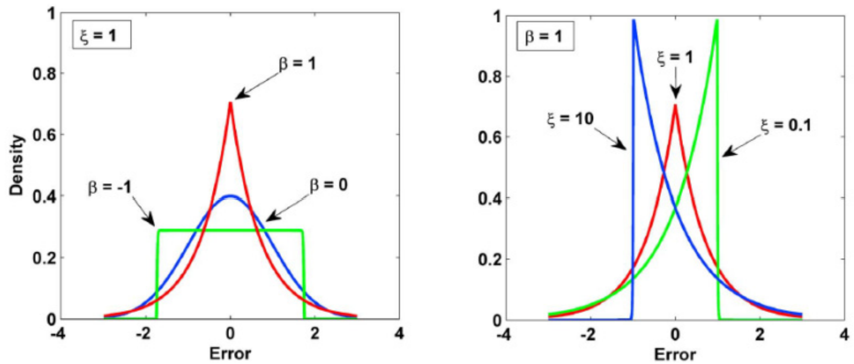
Modeling the Errors dependence
through an **AR(p)** model

Unbiased Studentized Error $\eta_t = \dfrac{e|q_{s,t} - b|q_{s,t}}{\sigma_{\varepsilon|q_{s,t}}} = \dfrac{\varepsilon|q_{s,t}}{\sigma_{\varepsilon|q_{s,t}}}$

$$\eta_t = \sum_{i=1}^{p} \phi_i \eta_{t-i} + \boxed{z_t} \equiv \phi_p(B)\eta_t = z_t$$

Innovations (White Noise)

Modeling innovations Z_t through the flexible Skew Exponential Distribution (**SEP**)



[Schoups and Vrugt (2010)]

Symmetric and Gaussian as particular cases

$$q_t = q_{s,t} + b|q_{s,t} + \sigma_{\varepsilon|q_{s,t}}\left[\phi_p^{-1}(B)[Z_t]\right] \rightarrow p\left(q_t|q_{s,t}\right) = p\left(e|q_{s,t}\right) = p\left(\varepsilon|q_{s,t}\right)$$

Phase I: Total Laws

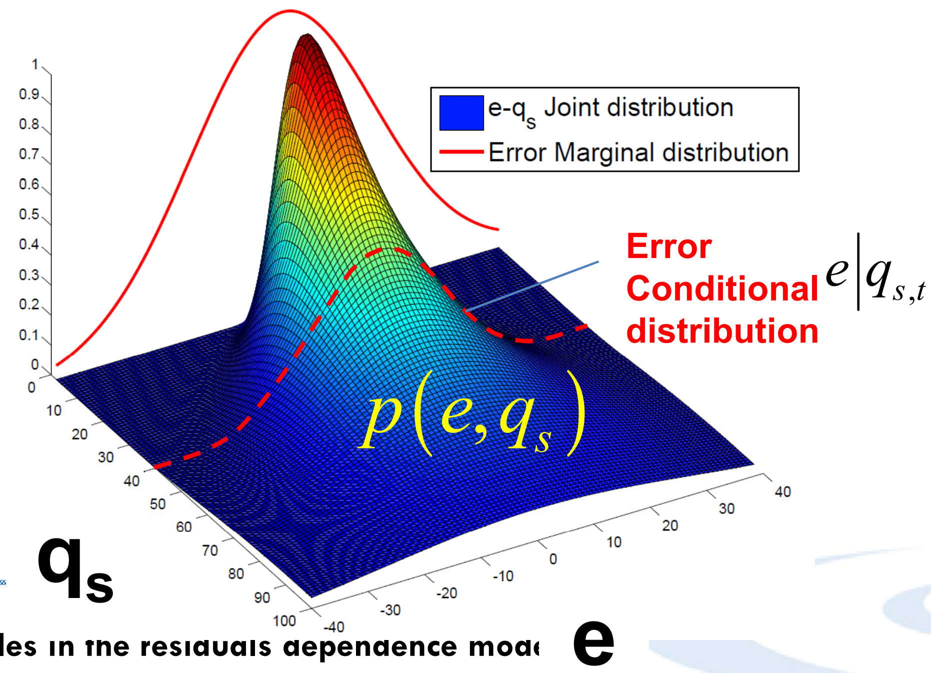
- **In defined Error model, some parameters of variance & Bias functions are not free !**
  - **Marginal and Conditional** Error distributions belong to the **same joint** distribution $p(e, q_s)$
    - Linked by Total Variance Law (**TVL**) and Total Expectation Law (**TEL**)
    - For the correct implementation of the **JOINT INFERENCE with a Time-Varying Error Model ➔ TOTAL LAWS** must be enforced **!**

TVL

$$V(\mathbf{e}) = E\left[V\left(e|q_{s,t}\right)\right] + V\left[E\left(e|q_{s,t}\right)\right]$$

TEL

$$E(\mathbf{e}) = E\left[E\left(e|q_{s,t}\right)\right] = E\left[b|q_{s,t}\right]$$



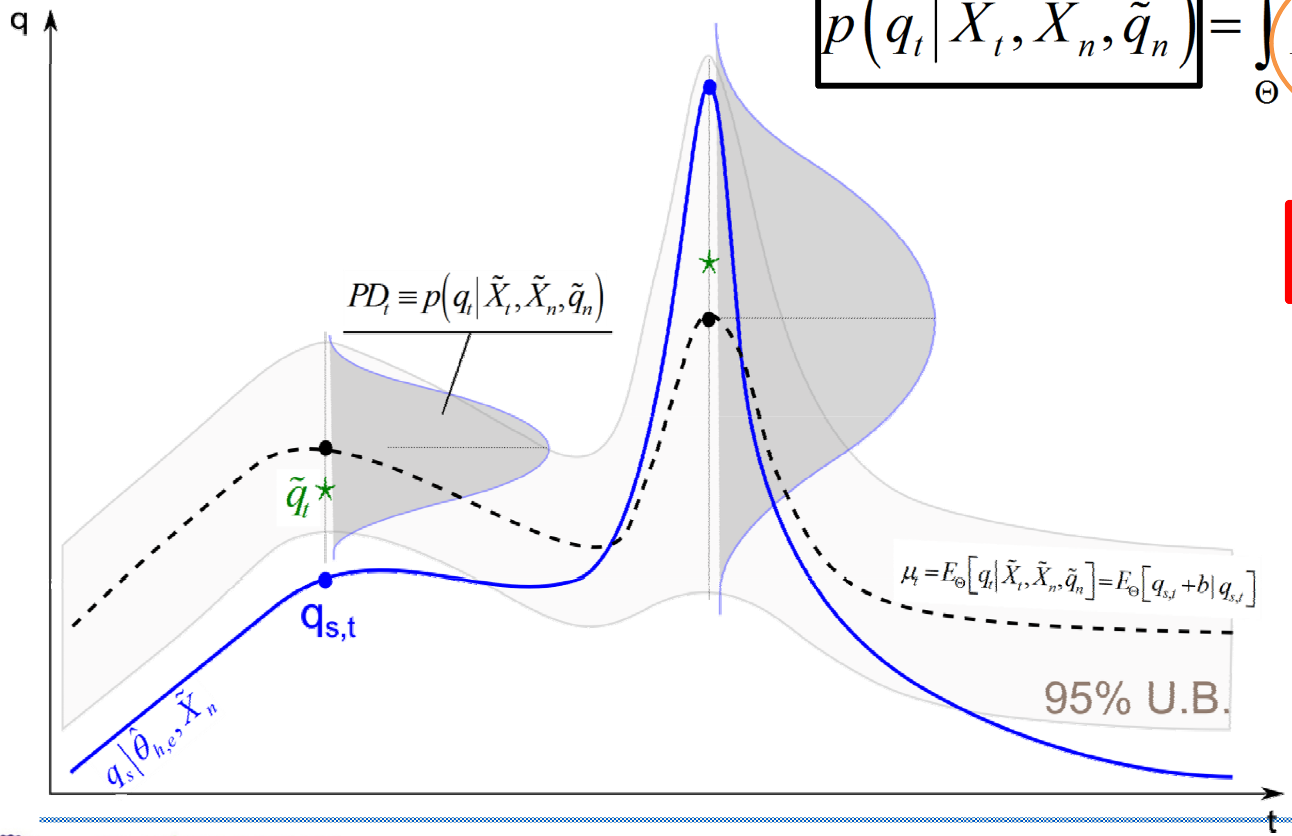
Phase I: Predictive uncertainty

Given the previously obtained, **Predictive pdf conditioned** on the simulated streamflow…

$$p\left(q_t | q_{s,t}\right) = p\left(e | q_{s,t}\right) = p\left(\varepsilon | q_{s,t}\right)$$

…we can get the **Predictive pdf** by its Marginalization on the parameters…

$$p\left(q_t | \tilde{X}_t, \tilde{X}_n, \tilde{q}_n\right) = \int_\Theta p\left(q_t | \theta_{h,e}, \tilde{X}_n\right) g\left(\theta_{h,e} | \tilde{q}_n, \tilde{X}_n\right) d\theta$$

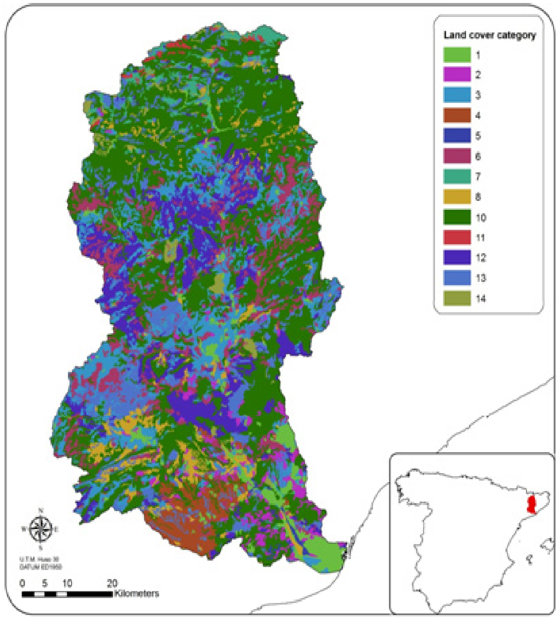


Posterior of parameters

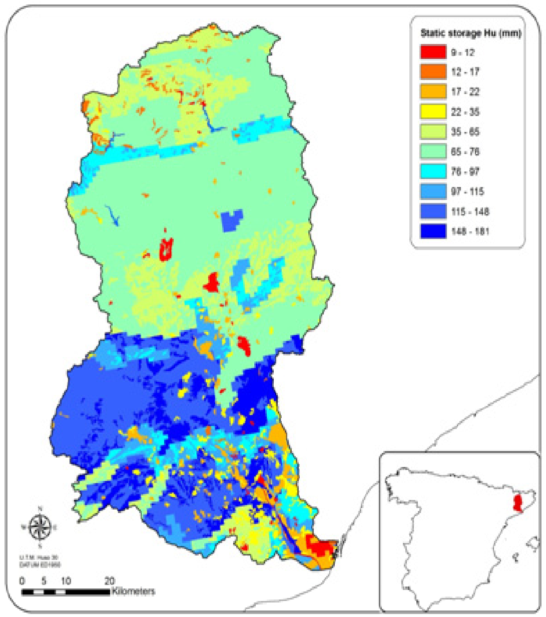
- Bayesian **Joint** inference
- MCMC sampling
  - ***DREAM-ZS*** algorithm
    [*Ter Braak and Vrugt (2008)*]

Phase I: Case Study

- **Distributed Hydrological Model (on a Spanish humid catch.)**
  - **TETIS** http://lluvia.dihma.upv.es/EN/software/software.html
  - ➢ Effective Parameter Structure divided in two parts:
    - An estimated Value in each cell setting-up the **Parameter Maps**
    - **Regularization Function**: **Global calibrated correction factor F_i** applied to each parameter map



x F1



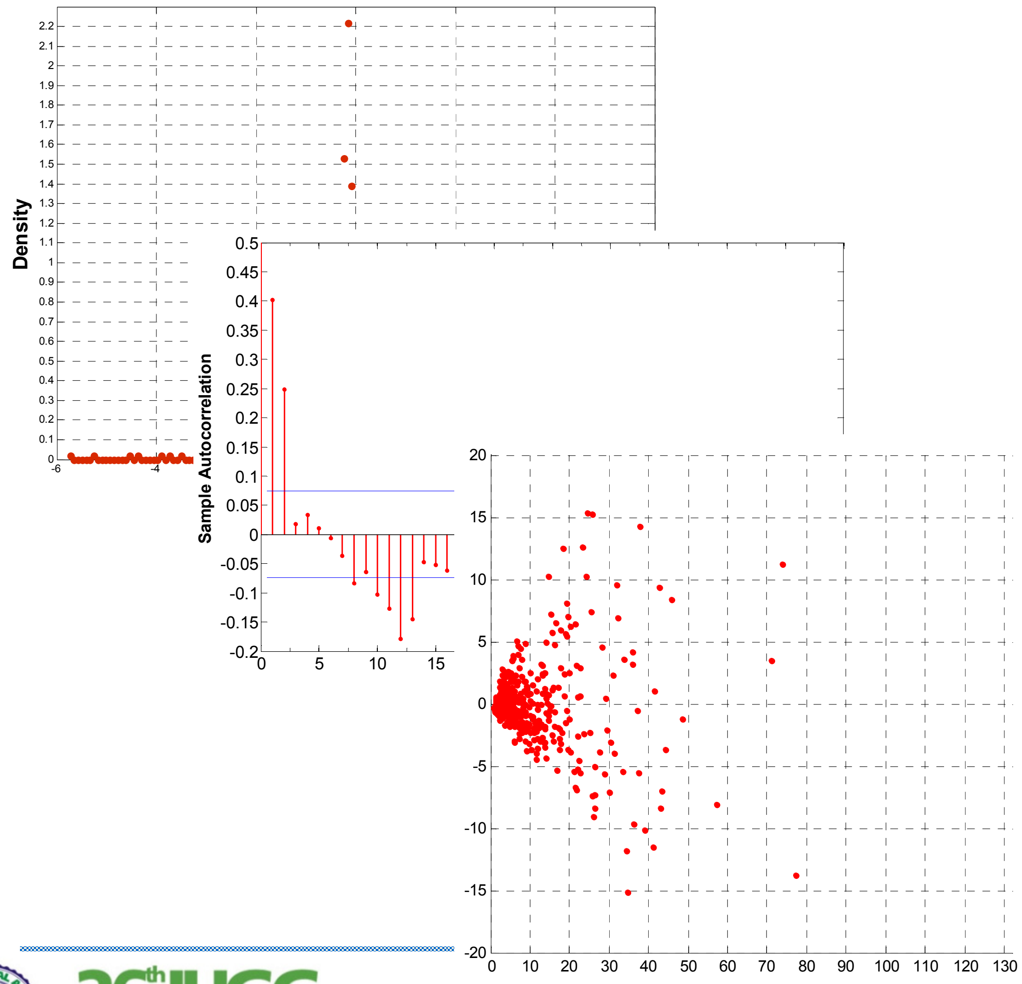
x F2 …….

Phase I: SLS vs EM2 comparison (1/5)

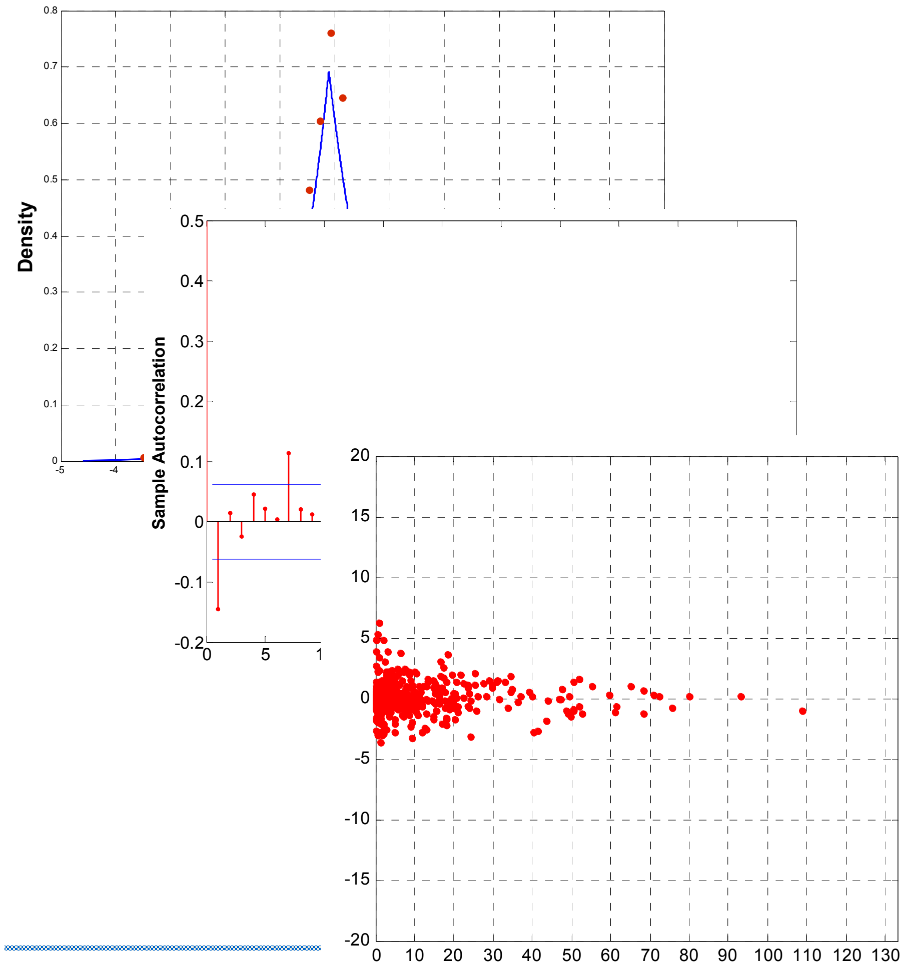
- **Fulfillment of the Error Model Hypothesis**

Normality ***Independence*** ***Homoscedasticity***

SLS



EM2



Inclusion of hydrological state variables in the residuals dependence model

Phase I: SLS vs EM2 comparison (2/5)

- **Simulation performance**
  - **In our Case Study** both show a **similar performance** of prediction in Validation based on NSE, RMSE, and VE% indexes

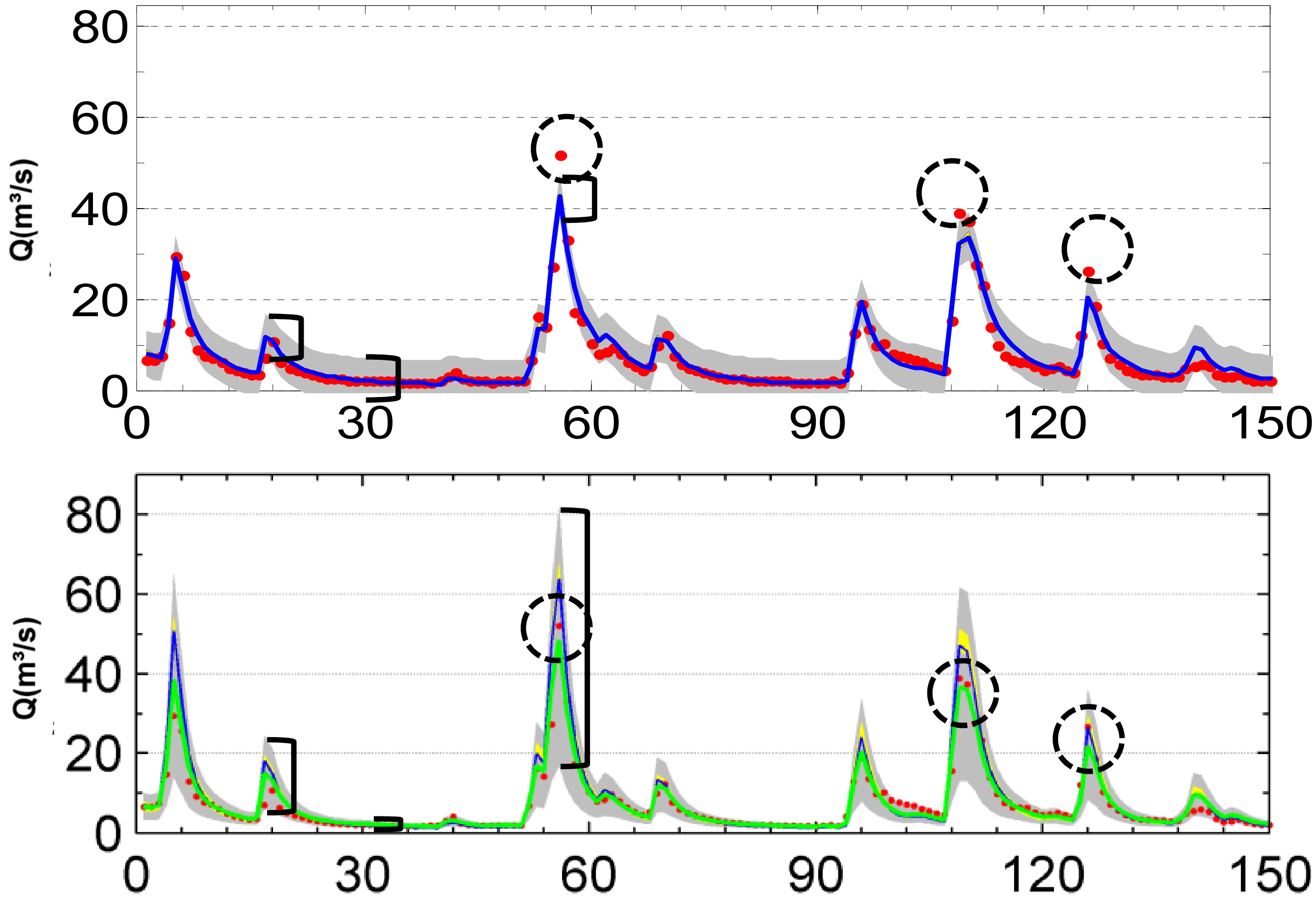
		SLS			EM2		
		CALIB	VALID	% CHANGE	CALIB	VALID	% CHANGE
HYDRO MODEL	NSE	0.93	0.86	7%	0.74	0.72	3%
	RMSE	2.62	3.48	33%	5.00	4.99	0%
	ErrVol (%)	2.40	-4.5	88%	9.90	2.70	73%
MEAN PREDICTION	NSE				0.91	0.85	7%
	RMSE				2.92	3.60	23%
	ErrVol (%)				0.01	-3.70	

Phase I: SLS vs EM2 comparison (3/5)

- **Assessment of the Predictive Uncertainty**
  - **95% Uncertainty Band**

SLS

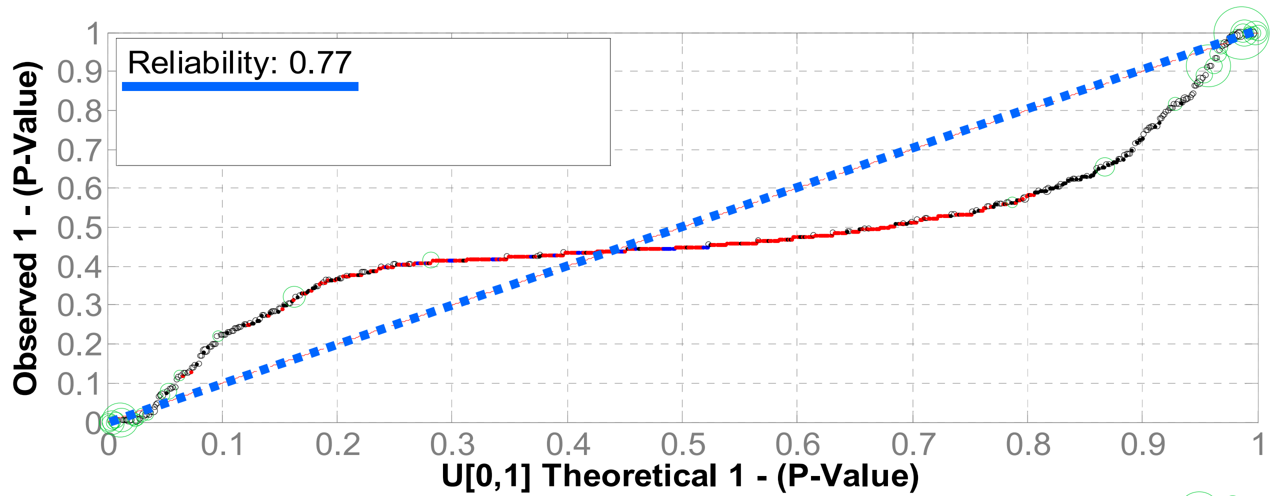
EM2



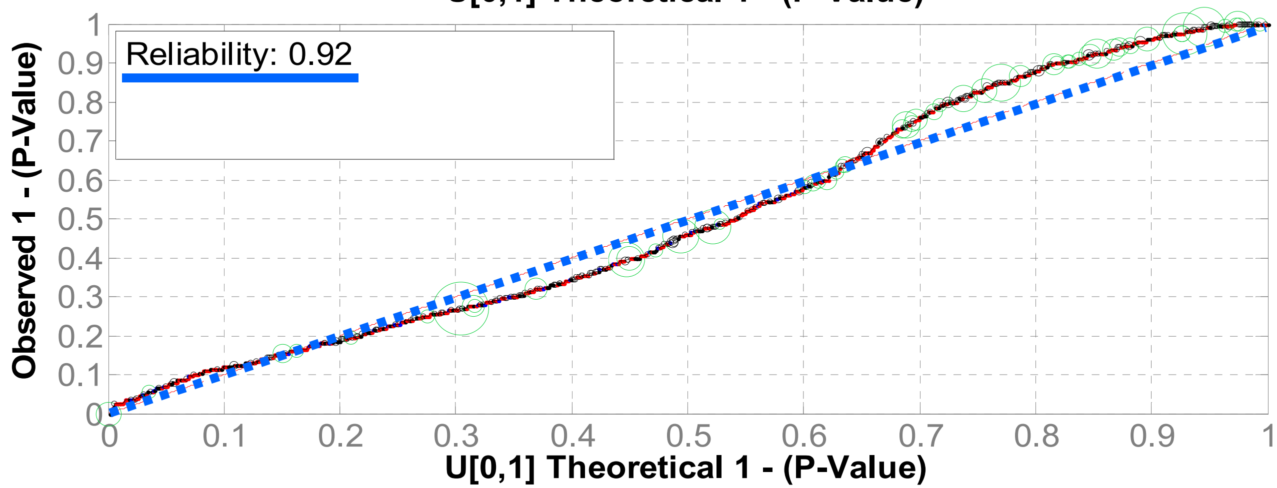
Phase I: SLS vs EM2 comparison (4/5)

- Assessment of the Predictive Uncertainty
  - Full Predictive distribution Reliability (PP-PLOTS)

SLS



EM2



Phase I: SLS vs EM2 comparison (5/5)

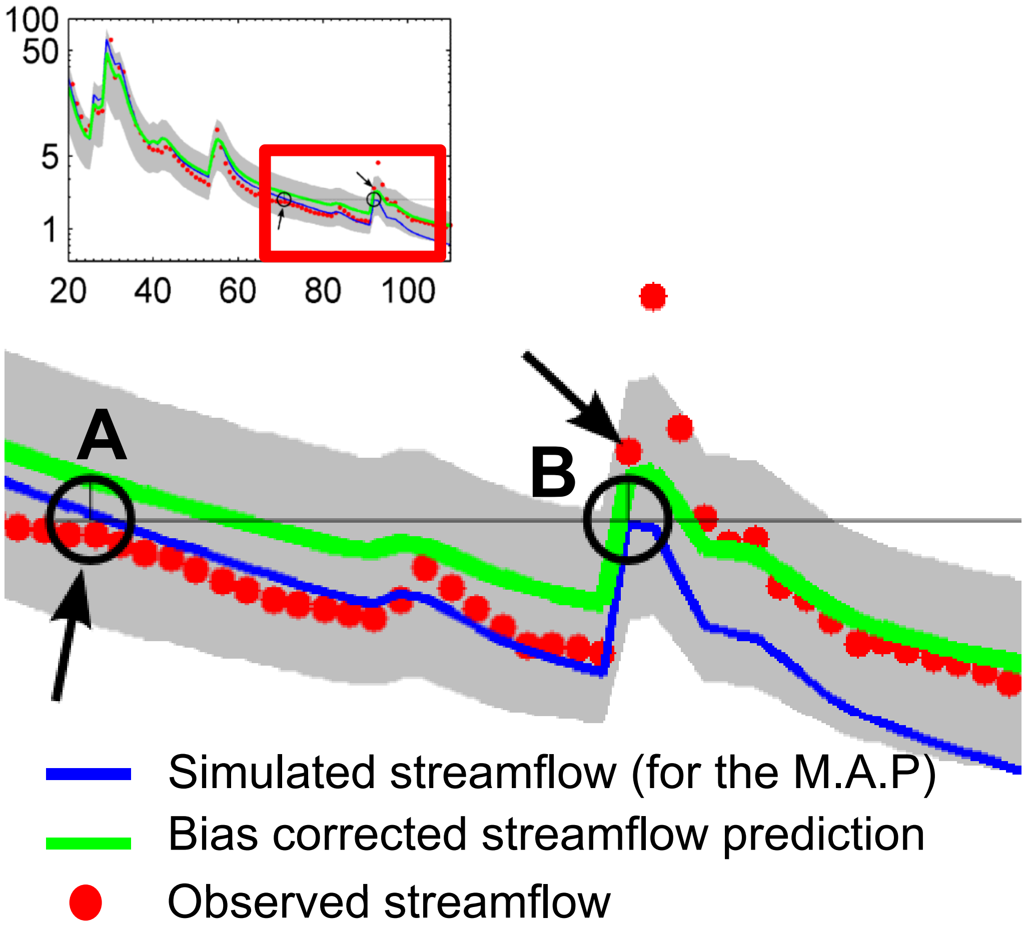
- **Parameters value coherence**
  - **EM2 shows less Biased parameters than SLS**
    - EM2 exhibits less deterioration of the H. Model performance between calibration and validation (**Divergence Phenomenon**)

		SLS			EM2		
		CALIB	VALID	% CHANGE	CALIB	VALID	% CHANGE
HYDRO MODEL	NSE	0.93	0.86	7%	0.74	0.72	3%
	RMSE	2.62	3.48	33%	5.00	4.99	0%
	ErrVol (%)	2.40	-4.5	88%	9.90	2.70	73%

- For some parameters SLS yields inferred values without **physical meaning**

Motive of Phase II

- **Phase I:** Conditional Bias ←==→ q_s **a biunivocal relation**
  - it doesn't show a good performance in validation



- Point **A**: at streamflow recession scenario
  Point **B**: at streamflow peak scenario
  - Both points show same "q_s" (**Blue L.**) and therefore same Bias (**Green L.**)
  - Observations $\tilde{q}_t$ (**Red points**)
    - At A, below "q_s"
    - At B, above "q_s"

> Different hydrological scenarios with different active processes, can yield **the same "q_s"** value

> But H. Model can produce **different Bias** when it models those different hydrological scenarios

Phase II: Bias Model description (1/2)

- In classical linear regression is common the analysis of residuals searching for a **misspecification of the fitted model** $e = f\left(regressors\right)$**?**
- A fitted H. model with **structural problems** could also exhibit **residuals correlated with "exogenous" variables (regressors)** of the modeled processes …
  - Variables to include ? (all Standardized $\mathbf{X}_k$ k=1,…,n):
    - **State Variables: Runoff** (t), **Interflow**(t) and **Base Flow** (t, t-1, t-2, …)
    - Or even, **Forcing: Precipitation** (t, t-1, t-2, …)
  - Residuals dependence model: **ARX(p,q)**

$$\eta_t = \sum_{i=1}^{p} \phi_i \eta_{t-i} + \sum_{k=1}^{n} \sum_{j=0}^{q} W_{kj} X_{k,t-j} + Z_{arx,t} \equiv \phi_p\left(B\right)\boldsymbol{\eta} = \sum_{k=1}^{n} W_{kq}\left(B\right)\mathbf{X}_k + \mathbf{Z}_{arx}$$

Studentized Error — Autoregressive Filter — "Exogenous" Filter — Innovations (White Noise)

Phase II: Bias Model description (2/2)

- **Relation between ARX(p , q) model and the Bias model**

$$\phi_p(B)\,\boldsymbol{\eta} = \sum_{k=1}^{n} W_{kq}(B)\,\mathbf{X}_k + \mathbf{Z}_{arx}$$

$$\boldsymbol{\eta} = \phi_p^{-1}(B)\left[\sum_{k=1}^{n} W_{kq}(B)\,\mathbf{X}_k + \mathbf{Z}_{arx}\right] = \underbrace{\phi_p^{-1}(B)\left[\sum_{k=1}^{n} W_{kq}(B)\,\mathbf{X}_k\right]}_{\boldsymbol{\eta}_I} + \underbrace{\phi_p^{-1}(B)\left[\mathbf{Z}_{arx}\right]}_{\boldsymbol{\eta}_{II}}$$

$\boldsymbol{\eta}_I$ Explained part of Studentized Error

$\boldsymbol{\eta}_{II}$ Random part of Studentized Error

$$\boldsymbol{\eta} = \frac{\mathbf{e}}{\boldsymbol{\sigma}_{e|q}}$$

Studentized Error

$$\boldsymbol{\eta} = \frac{\mathbf{e}}{\boldsymbol{\sigma}_{e|q}} = \boldsymbol{\eta}_I + \boldsymbol{\eta}_{II}$$

$$\frac{\mathbf{e} - \boldsymbol{\eta}_I \boldsymbol{\sigma}_{e|q}}{\boldsymbol{\sigma}_{e|q}} = \boldsymbol{\eta}_{II}$$

UNBIASED Studentized Error

$$E[\boldsymbol{\eta}_{II}] = 0 \qquad V[\boldsymbol{\eta}_{II}] = 1$$

- $\boldsymbol{\eta}_I \boldsymbol{\sigma}_{e|q}$: Bias model is a function of "n" $\mathbf{X}_k$ **Std. State Variables**
  - $\mathbf{W}_{kq}$ { are regression coefficients to be **inferred JOINTLY**; offer us keys about the **sources of the Bias** (research!)

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Phase II: *"Man is the only animal to stumble twice on the same stone"* *Spanish proverb*

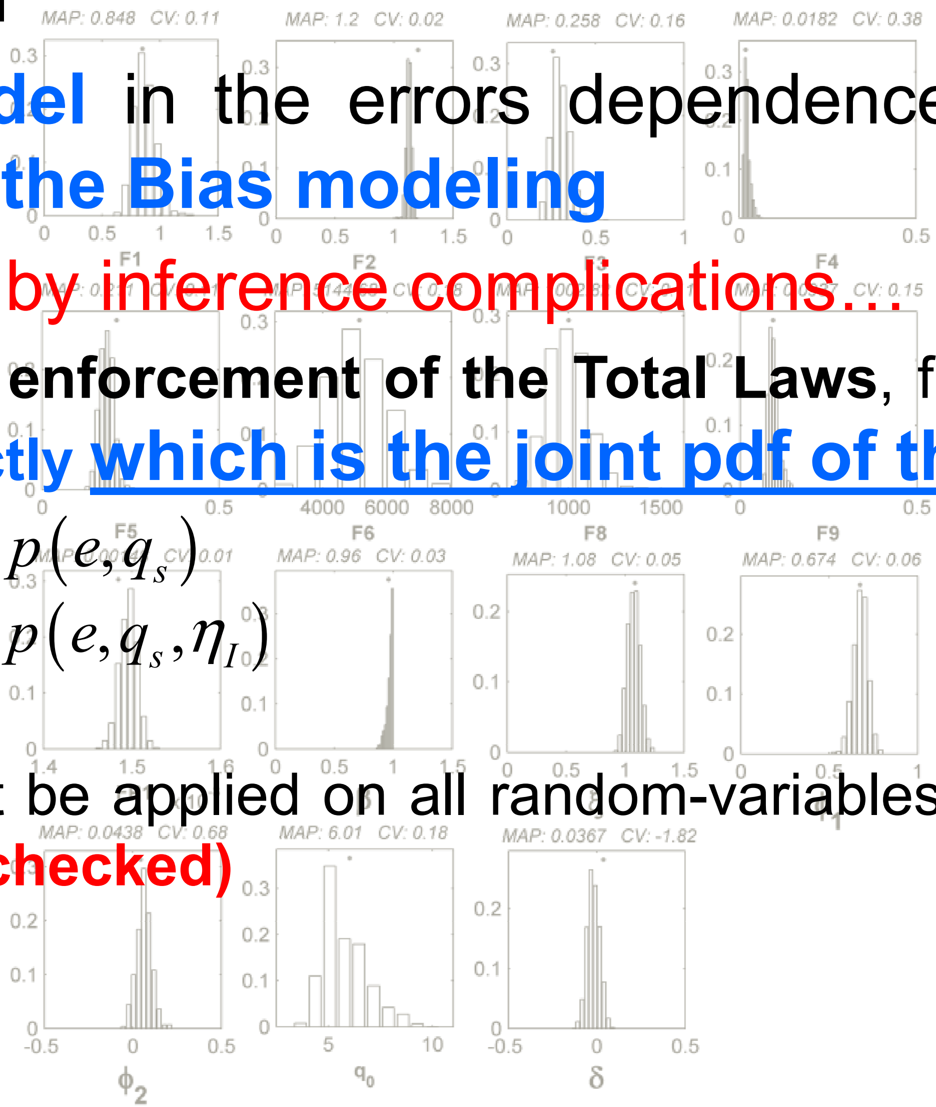
- **Inference complications in Phase II**
  - Markov chains do not converge (> 1.000.000 Iter.)
  - Inferred lag(1) autocorrelation parameter too high (~ 0.99)
  - Similar problems reported in papers where **TL** were not applied (e.g. [*Schoups and Vrugt (2010)*] [Evin et al. *(2013, 2014)*])
- **Suspected origin of the problem:**
  - **In Phase I** we had a **bivariate pdf** $p(e, q_s)$ on which **TL** were applied, and inference was **successful**
  - **In Phase II** we have a **trivariate pdf** $p(e, q_s, \eta_I)$ on which **TL** is applied but only on $p(e, q_s)$ and inference has **failed…**
    - ***Total Laws*** must be enforced also,
      - on $p(e, \eta_I)$ and on $p(q_s, \eta_I)$ **?**

Conclusions

- For the **Phase I**
  - In our **Case Study**, Error models performance shows that
    - SLS doesn't fulfill **the initial hypotheses** while the EM2 fulfillment is good
    - **Simulations in validation** show that EM2 and SLS have a **similar performance** (**NSE** & **Vol.Error**)
    - EM2 yields **less biased hydrological parameters**, which is interesting for Regionalization methods
    - EM2 shows **better** assessment of the **Predictive Uncertainty**
  - **Bias model** needs to consider **more explicative variables** than the simulated streamflow → Motive of Phase II
  - Time-Varying Error Models must enforce ***THE TOTAL LAWS*** (**TVL** and **TEL**)

Conclusions

- For the **Phase II**
  - An **ARX model** in the errors dependence model, it is proposed **for the Bias modeling**
  - Unconcluded by inference complications…
    - It seems that **enforcement of the Total Laws**, firstly requires to **define correctly which is the joint pdf of the Error**
      - Bivariate $p(e,q_s)$
      - Trivariate $p(e,q_s,\eta_I)$
      - …
    - If so, TL must be applied on all random-variables in that joint pdf **(issue to be checked)**



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Thank you

for your attention

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